

11.3 Sum and Difference Identities

Write your questions here!



Is it true?

$$\sin(45^\circ + 30^\circ) = \sin 45^\circ + \sin 30^\circ$$

Sum/Difference Identities

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \sin \beta \cos \alpha$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Ex 1:

$$\cos 15$$

Ex 2:

$$\sin 105$$

Ex 3: $\tan \frac{11\pi}{12}$

Ex 4: Write the expression as the sine, cosine, or tangent of an angle.

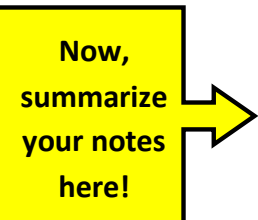
$$\sin 42 \cos 17 - \cos 42 \sin 17$$

Ex 5: Find $\sin(x - y)$ given the following:

$$\cos x = -\frac{3}{5}, \text{ where } \frac{\pi}{2} < x < \pi \text{ and } \tan y = \frac{8}{15}, \text{ where } \frac{3\pi}{2} < y < 2\pi$$

Ex 6: Is the equation an identity? $\tan(x - \pi) = \tan x$

SUMMARY:



11.3 Sum and Difference Identities

PRACTICE

Directions: Tell whether each statement is true or false.

1) $\sin 75 = \sin 50 \cos 25 - \cos 25 \sin 25$

2) $\cos 15 = \cos 60 \cos 45 + \sin 60 \sin 45$

3)
$$\tan 225 = \frac{\tan 180 - \tan 45}{1 + \tan 180 \tan 45}$$

Directions: Write the expression as the sine, cosine or tangent of an angle.

4) $\sin 42 \cos 17 - \cos 42 \sin 17$

5)
$$\frac{\tan 19 + \tan 47}{1 - \tan 19 \tan 47}$$

6)
$$\cos \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{3} \sin \frac{\pi}{4}$$

Directions: Use the sum or difference identity to find the exact value.

7) $\tan 195^\circ$

8) $\cos 255^\circ$

9) $\sin 165^\circ$

10) $\cos \frac{13\pi}{12}$

$$11) \sin \frac{5\pi}{12}$$

$$12) \tan \frac{\pi}{12}$$

Directions: Find the exact value.

$$13) \sin(\alpha - \beta)$$

$$\text{Given: } \cos \alpha = \frac{3}{5}, \text{ where } 0 < \alpha < \frac{\pi}{2}$$

$$\tan \beta = \frac{12}{5}, \text{ where } 0 < \beta < \frac{\pi}{2}$$

$$14) \tan(x - y)$$

$$\text{Given: } \cos x = \frac{7}{25}, \text{ where } 0^\circ < x < 90^\circ$$

$$\cos y = -\frac{4}{5}, \text{ where } 90^\circ < y < 180^\circ$$

$$15) \sin(\alpha + \beta)$$

$$\text{Given: } \sin \alpha = \frac{4}{5}, \text{ where } \alpha \text{ is in Quadrant I}$$

$$\cos \beta = -\frac{24}{25}, \text{ where } \beta \text{ is in Quadrant III}$$

$$16) \cos(x + y)$$

$$\text{Given: } \cos x = \frac{15}{17}, \text{ where } \frac{3\pi}{2} < x < 2\pi$$

$$\tan y = \frac{4}{3}, \text{ where } \pi < y < \frac{3\pi}{2}$$

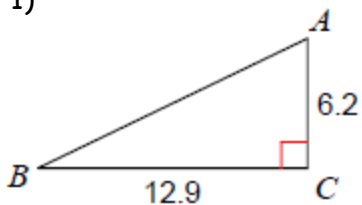
Directions: Is the equation an identity? Explain using the sum or difference identities

17) $\cos(x - \pi) = \cos x$

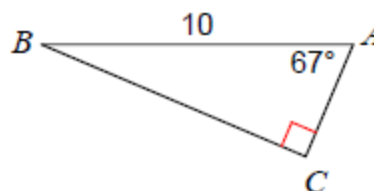
18) $\sin(x - \pi) = \sin x$

REVIEW SKILLZ: Directions: Solve each triangle.

1)



2)



11.3 Application and Extension

1) Find the exact value.

$\cos 285^\circ$

2) Find the exact value.

$\cos(x + y)$

Given: $\cos x = \frac{15}{17}$, where $\frac{3\pi}{2} < x < 2\pi$

$\tan y = \frac{4}{3}$, where $\pi < y < \frac{3\pi}{2}$

3) Verify the following DOUBLE ANGLE IDENTITIES. (Hint.... $\sin(2x) = \sin(x + x)$)

a) $\sin(2x) = 2 \sin x \cos x$

b) $\cos(2x) = 2 \cos^2 x - 1$

5) When a wave travels through a taut string (like guitar string), the displacement y of each point on the string depends on the time t and the point's position x . The equation of a standing wave can be obtained by adding the displacements of two waves traveling in opposite directions. Suppose two waves can be modeled by the following equations:

$$y_1 = A \cos\left(\frac{2\pi t}{3} - \frac{2\pi x}{5}\right)$$

$$y_2 = A \cos\left(\frac{2\pi t}{3} + \frac{2\pi x}{5}\right)$$

Find $y_1 + y_2$