11.4 Double and Half Angle Identities

NOTES

Write your questions here!

$\sin 2\theta$ $\cos 2\theta$ $\tan 2\theta$

Ex 1: If $\cos\theta = \frac{\sqrt{5}}{5}$ and $0° < \theta < 90°$, find the exact value of each function.

$\sin 2\theta$ $\cos 2\theta$

Ex 2: Find the exact value of each!

$\sin 22.5$ $\cos\frac{7\pi}{8}$

Ex 3: If $\sin\theta = \frac{5}{7}$ and $90° < \theta < 180°$, find the exact value of each function.

$\tan 2\theta$ $\tan\frac{\theta}{2}$

Ex 4: Verify the following: $\cot x = \frac{\sin 2x}{1-\cos 2x}$

Ex 5: Verify the following: $\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$

Now, summarize your notes here!

SUMMARY:

11.4 Double and Half Angle Identities

PRACTICE

Directions: Tell whether each statement is true.

1) $\cos 2(20°) = 2\cos^2 40° - 1$

2) $\cos(70°) = \cos^2 35° - \sin^2 35°$

3) $\tan\frac{140°}{2} = -\sqrt{\frac{1-\cos 140°}{1+\cos 140°}}$

Directions: Find the exact value of the given function.

4) $\cos 75°$

5) $\sin\frac{5\pi}{8}$

Directions: For #6-9: If $\sin x = \frac{3}{5}$ and x is in Quadrant II, find each value. Draw the reference triangle.

6) $\cos 2x$

7) $\tan 2x$

8) $\sin\frac{x}{2}$

9) $\cos\frac{x}{2}$

Directions: For #10-13: If $\cos\theta = -\frac{1}{3}$ and x is in Quadrant II, find each value. Draw the reference triangle.

10) $\cos 2\theta$

11) $\sin 2\theta$

12) $\tan\frac{\theta}{2}$

13) $\sin\frac{\theta}{2}$

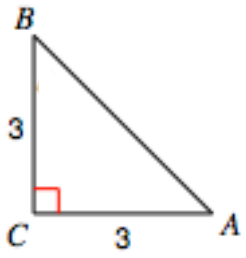
Directions: Verify the following identities.

14) $1 + \sin 2x = (\sin x + \cos x)^2$

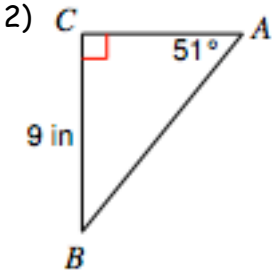
15) $\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$

REVIEW SKILLZ: Directions: Solve the triangle.

1)



2)



11.4 Application and Extension

If $\tan\theta = -\frac{1}{3}$ and x is in Quadrant II, find each value. Draw the reference triangle.

1) $\cos 2x$

2) $\tan\frac{x}{2}$

3) Mr. Allen is a tremendous golfer. He can hit the ball with an initial velocity of 50 feet per second. The distance that a golf ball travels is found by the formula $d = \frac{v_0}{g}\sin 2\theta$, where v_0 is the initial velocity, g is the acceleration due to gravity and θ is the measure of the angle that the inititial path of the ball makes with the ground. The acceleration due to gravity is 32 ft/s^2.

a. Write an expression for the distance the ball travels in terms of θ.

b. Use a calculator to find the distance Mr. Allen's ball traveled if the angle between the initial path of the ball and the ground measured $60°$.

4) For each equation try the following values $\left(x = 0, \pi, \frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{6}\right)$ and calculate both the left and right sides of the equation. If the equation is an identity, VERIFY IT! If it is not an identity, find a value of x for which both sides are defined but not equal.

a) $\sin 4x = 4 \sin x \cos x$

b) $\sin 2x = (\tan x)(1 + \cos 2x)$

BRING THE PAIN! Use your graphing calculator and determine which of the following three equations is a trig identity. Then verify it!

$\tan 2x = \frac{2}{\tan x - \cot x}$

$\cot 2x = \frac{\tan x(cot^2 x - 1)}{2}$

$sin^2 x = \frac{1 - \cos 2x}{2}$