11.4 Double and Half Angle Identities

PRACTICE

Directions: Tell whether each statement is true.

1) $\cos 2(20°) = 2\cos^2 40° - 1$

FALSE – should be $= 2\cos^2(20°) - 1$

2) $\cos(70°) = \cos^2 35° - \sin^2 35°$

TRUE

3) $\tan\frac{140°}{2} = -\sqrt{\frac{1-\cos 140°}{1+\cos 140°}}$

False...half of 140 is 70 and that is in the first quadrant. Tangent is positive in the first quadrant.

Directions: Find the exact value of the given function.

4) $\cos 75° = \cos\left(\frac{150}{2}\right)$

$= \pm\sqrt{\frac{1+\cos 150}{2}} = \pm\sqrt{\frac{1+\frac{-\sqrt{3}}{2}}{2}}$

$= \pm\sqrt{\frac{\frac{2}{2}+\frac{-\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$

$\boxed{= +\sqrt{\frac{2-\sqrt{3}}{4}}}$

* Positive because 75° is in QUAD I.

5) $\sin\frac{5\pi}{8} = \sin\left(\frac{5\pi}{4}\cdot\frac{1}{2}\right)$

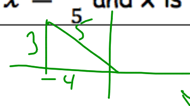
$= \pm\sqrt{\frac{1-\cos\left(\frac{5\pi}{4}\right)}{2}} = \pm\sqrt{\frac{1-\frac{-\sqrt{2}}{2}}{2}}$

$= \pm\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}} = \pm\sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}}$

$\boxed{= +\sqrt{\frac{2+\sqrt{2}}{4}}}$

* Positive because half of $\frac{5\pi}{4}$ is in QUAD II.

Directions: For #6-9: If $\sin x = \frac{3}{5}$ and x is in Quadrant II, find each value. Draw the reference triangle.



6) $\cos 2x$

$= \cos^2 x - \sin^2(x)$

$= \left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$\boxed{= \frac{7}{25}}$

Same

7) $\tan 2x$

$= \frac{2\tan x}{1-\tan^2 x}$

$= \frac{2\left(\frac{-3}{4}\right)}{1-\left(\frac{-3}{4}\right)^2} = \frac{-\frac{3}{2}}{1-\frac{9}{16}}$

$= \frac{-\frac{3}{2}}{\frac{7}{16}} = -\frac{3}{2}\left(\frac{16}{7}\right)$

$\boxed{= \frac{-24}{7}}$

8) $\sin\frac{x}{2}$

$90 < x < 180$
$45 \quad 90$
so sin is positive

$= \pm\sqrt{\frac{1-\cos x}{2}}$

$= +\sqrt{\frac{1-\left(\frac{-4}{5}\right)}{2}} = \sqrt{\frac{\frac{5}{5}+\frac{4}{5}}{2}}$

$= \sqrt{\frac{\frac{9}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{\sqrt{9}}{\sqrt{10}} = \frac{3}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}}$

$\boxed{= \frac{3\sqrt{10}}{10}}$

9) $\cos\frac{x}{2}$

$90 < x < 180$
$45 < x < 90$
so cos is positive

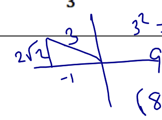
$= \pm\sqrt{\frac{1+\cos x}{2}}$

$= \sqrt{\frac{1+\left(\frac{-4}{5}\right)}{2}} = \sqrt{\frac{\frac{5}{5}-\frac{4}{5}}{2}}$

$= \sqrt{\frac{\frac{1}{5}}{2}} = \sqrt{\frac{1}{5}\left(\frac{1}{2}\right)} = \sqrt{\frac{1}{10}}$

$= \frac{\sqrt{1}}{\sqrt{10}} = \frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} \leftarrow \boxed{\frac{\sqrt{10}}{10}}$

Directions: For #10-13: If $\cos\theta = -\frac{1}{3}$ and x is in Quadrant II, find each value. Draw the reference triangle.



$3^2 = (-1)^2 + y^2$

$9 - 1 \leq y^2$

$(8 = y^2)$

$\sqrt{4\sqrt{2}} = y$

$2\sqrt{2}$

10) $\cos 2\theta$

$= \cos^2\theta - \sin^2\theta$

$= \left(\frac{-1}{3}\right)^2 - \left(\frac{2\sqrt{2}}{3}\right)^2$

$= \frac{1}{9} - \frac{4 \cdot 2}{9}$

$= \frac{1-8}{9} = \boxed{\frac{-7}{9}}$

11) $\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\frac{2\sqrt{2}}{3}\right)\left(\frac{-1}{3}\right)$

$\boxed{= \frac{-4\sqrt{2}}{9}}$

12) $\tan\frac{\theta}{2}$

$\cancel{90 < x < 180}$

$45 < \frac{x}{2} < 90$

so tan is positive

$= \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

$= \sqrt{\frac{1-\left(\frac{-1}{3}\right)}{1+\left(\frac{-1}{3}\right)}} = \sqrt{\frac{1+\frac{1}{3}}{1-\frac{1}{3}}}$

$= \sqrt{\frac{\frac{3}{3}+\frac{1}{3}}{\frac{3}{3}-\frac{1}{3}}} = \sqrt{\frac{\frac{4}{3}}{\frac{2}{3}}} = \sqrt{\frac{4}{3}\left(\frac{3}{2}\right)}$

$\boxed{= \sqrt{2}}$

13) $\sin\frac{\theta}{2}$

$90 < x < 180$

$45 < \frac{x}{2} < 90$

so sin is positive

$\pm\sqrt{\frac{1-\cos x}{2}}$

$= \sqrt{\frac{1-(-1/3)}{2}}$

$= \sqrt{\frac{\frac{3}{3}+1/3}{2}} = \sqrt{\frac{\frac{4}{3}}{2}} = \sqrt{\frac{4}{6}}$

$= \sqrt{\frac{2}{3}} = \frac{\sqrt{2}\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \boxed{\frac{\sqrt{6}}{3}}$

Directions: Verify the following identities.

14) $1 + \sin 2x = (\sin x + \cos x)^2$

$= (\sin x + \cos x)(\sin x + \cos x)$

$= \sin^2 x + \sin x\cos x + \sin x\cos x + \cos^2 x$

$= \underbrace{\sin^2 x + \cos^2 x}_{1} + 2\sin x\cos x$

$1 + 2\sin x\cos x \quad = 1 + 2\sin x\cos x$ ✓ ✓

*sub in double angle

*sub in $s^2 + c^2 = 1$

15) $\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$

$\sqrt{\frac{1-\cos x}{1-\cos x}}\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x}\frac{(1-\cos x)}{(1-\cos x)}$

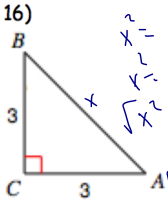
$\sqrt{\frac{(1-\cos x)^2}{1-\cos^2 x}} = \frac{\sin x(1-\cos x)}{1-\cos^2 x}$

$\sqrt{\frac{(1-\cos x)^2}{\sin^2 x}} = \frac{\sin x(1-\cos x)}{\sin^2 x}$

✓ $\frac{1-\cos x}{\sin x} = \frac{1-\cos x}{\sin x}$ ✓

Directions: Solve the triangle.

16)



$x^2 = 3^2 + 3^2$

$x^2 = 9 + 9$

$\sqrt{x^2} = \sqrt{18}$

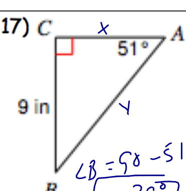
$= \sqrt{9}\sqrt{2}$

$\boxed{x = 3\sqrt{2}}$

$\boxed{\angle A = 45^\circ}$

$\boxed{\angle B = 45^\circ}$

17)



$\angle B = 90 - 51$

$\boxed{= 39^\circ}$

$\tan 51 = \frac{9}{x}$

$x = \frac{9}{\tan 51}$

$x = 7.3$

$\sin 51 = \frac{9}{y}$

$y = \frac{9}{\sin 51}$

$y = 11.6$