11.5 Trigonometric Equations

Directions: Find all exact solutions for $0 \leq x \leq 2\pi$.

1) $3\sec^2 x - 4 = 0$

$+4 \quad +4$

$\dfrac{3\sec^2 x}{3} = \dfrac{4}{3}$

$\sqrt{\sec^2 x} = \sqrt{\dfrac{4}{3}}$

$\sec x = \dfrac{\sqrt{4}}{\sqrt{3}} = \pm\dfrac{2}{\sqrt{3}}$

$\cos x = \pm\dfrac{\sqrt{3}}{2} = \boxed{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \text{ or } \dfrac{11\pi}{6}}$

2) $4\cos^2 x - 2 = 0$

$+2 \quad +2$

$\dfrac{4\cos^2 x}{4} = \dfrac{2}{4}$

$\sqrt{\cos^2 x} = \sqrt{\dfrac{1}{2}}$

$\cos x = \dfrac{\sqrt{1}}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\dfrac{\sqrt{2}}{\sqrt{2}}$

$\cos x = \pm\dfrac{\sqrt{2}}{2} = \boxed{\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \text{ or } \dfrac{7\pi}{4}}$

3) $2\sin^2 x + 5\sin x = 3$

$2\sin^2 x + 5\sin x - 3 = 0$

$(2\sin x - 1)(2\sin x + 6) = 0$

$(2\sin x - 1)(\sin x + 3) = 0$

$2\sin x - 1 = 0 \qquad \sin x + 3 = 0$
$\quad +1 \quad +1 \qquad\qquad -3 \quad -3$
$\dfrac{2\sin x}{2} = \dfrac{+1}{2} \qquad \cancel{\sin x = -3}$

$\sin x = \dfrac{1}{2}$

$\boxed{x = \dfrac{\pi}{6} \text{ or } \dfrac{5\pi}{6}}$

4) $4 = \dfrac{1+\sin x}{\cos x} + \dfrac{\cos x}{1+\sin x}$

$4 = \dfrac{(1+\sin x)^2 + \cos^2 x}{\cos x(1+\sin x)} \Rightarrow \quad 4 = \dfrac{1 + 2\sin x + \sin^2 x + 1 - \sin^2 x}{\cos x(1+\sin x)}$

$4 = \dfrac{2 + 2\sin x}{\cos x(1+\sin x)} \Rightarrow \quad 4 = \dfrac{2(1+\sin x)}{\cos x(1+\sin x)}$

$\cos x \cdot 4 = \dfrac{2}{\cos x}\cos x \Rightarrow \quad \dfrac{4\cos x}{4} = \dfrac{2}{4}$

$\cos x = \dfrac{1}{2} \qquad \boxed{x = \dfrac{\pi}{3} \text{ or } \dfrac{5\pi}{3}}$

Directions: Find all exact solutions for $0° \leq x \leq 360°$.

5) $\cos^2 x = 1 - \sin x$

$1 - \sin^2 x = 1 - \sin x$

$-\sin^2 x = -\sin x$

$0 = \sin^2 x - \sin x$

$0 = \sin x(1 - \sin x)$

$\sin x = 0 \qquad\qquad 1 - \sin x = 0$
$\boxed{x = 0°, 180°, 360°} \qquad 1 = \sin x$
$\qquad\qquad\qquad\qquad \boxed{x = 90°}$

6) $3\cot^2 x - 1 = 0$

$+1 \quad +1$

$\dfrac{3\cot^2 x}{3} = \dfrac{1}{3}$

$\sqrt{\cot^2 x} = \sqrt{\dfrac{1}{3}}$

$\cot x = \pm\dfrac{\sqrt{1}}{\sqrt{3}}$

$\cot x = \pm\dfrac{1}{\sqrt{3}} \Rightarrow \tan x = \pm\sqrt{3}$

$\boxed{x = 60°, 120°, 240° \text{ or } 300°}$

7) $\sin x - 2\sin x \cos x = 0$

$\sin x(1 - 2\cos x) = 0$

$\sin x = 0 \qquad\qquad 1 - 2\cos x = 0$
$\boxed{x = 0°, 180° \text{ or } 360°} \quad \dfrac{-2\cos x}{-2} = \dfrac{-1}{-2}$

$\qquad\qquad\qquad\qquad \cos x = \dfrac{1}{2}$

$\boxed{x = 60°, \text{ or } 300°}$

8) $\tan x = -2\sin x$

$\tan x + 2\sin x = 0$

$\dfrac{\sin x}{\cos x} + 2\sin x\left(\dfrac{\cos x}{\cos x}\right) = 0$

$\dfrac{\sin x + 2\sin x\cos x}{\cos x} = 0 \qquad \dfrac{\sin x}{\cos x}(1 + 2\cos x) = 0$

$\dfrac{\sin x(1 + 2\cos x)}{\cos x} = 0 \qquad\qquad 1 + 2\cos x = 0$

$\qquad\qquad \tan x = 0 \qquad\qquad 2\cos x = -1$
$\qquad\qquad \boxed{x = 0°, 180°} \qquad\qquad \cos x = -\dfrac{1}{2}$

$\qquad\qquad\qquad\qquad\qquad\qquad \boxed{x = 120°, 240°}$

Directions: Approximate all solutions for $0° \leq x \leq 360°$.

9) $10 \cos x - 4 = 4 \cos x$

$\underline{-4\cos x \qquad -4\cos x}$

$6\cos x - 4 = 0$

$6\cos x = 4$

$\cos x = \frac{4}{6} = \frac{2}{3}$

$x = \cos^{-1}\left(\frac{2}{3}\right) = \boxed{48.2°}$

$360 - 48.2 = \boxed{311.8°}$

10) $5 \sin^2 x + 3 \sin x = 1$ $a=5, b=3, c=-1$

$5\sin^2 x + 3\sin x - 1 = 0$

$\sin x = \frac{-3 \pm \sqrt{3^2 - 4(5)(-1)}}{2(5)} = \frac{-3 \pm \sqrt{29}}{10}$

$\sin x = .2385$ or $\sin x = -.8385$

$\boxed{x = 13.8° \text{ or } 166.2°}$

$x = -57° \Rightarrow$

$\boxed{237° \text{ or } 303°}$

11) $6 \sin^2 x + (1 - \cos^2 x) = 2$

$6\sin^2 x + \sin^2 x = 2$

$\frac{7\sin^2 x}{7} = \frac{2}{7}$

$\sqrt{\sin^2 x} = \sqrt{\frac{2}{7}} = \sin x = \pm\sqrt{\frac{2}{7}}$

$\sin x = \sqrt{\frac{2}{7}} = 32.3°$ $\sin x = -\sqrt{\frac{2}{7}} = -32.3$

$\boxed{x = 32.3°, 147.7°}$ $\boxed{212.3° \text{ or }, 327.2}$

12) $\dfrac{16 \tan^2 x}{16} = \dfrac{5}{16}$

$\sqrt{\tan^2 x} = \sqrt{\frac{5}{16}}$

$\tan x = \frac{\sqrt{5}}{4}$ or $\tan x = \frac{-\sqrt{5}}{4}$

$29.1°$ $= -29.2°$

$\boxed{x = 29.1°, 209.1°, 150.8°, \text{ or } 330.2°}$

Directions: Find all exact solutions.

13) $2 \cos^2 x + \cos x = 0$

$\cos x(2\cos x + 1) = 0$

$\cos x = 0$ $2\cos x + 1 = 0$

                $\cos x = -\frac{1}{2}$

$x = \boxed{90° + 360k}$ $\boxed{x = 120° + 360k}$

$\boxed{270° + 360k}$ $\boxed{240° + 360k}$

14) $3 \sin x = 2 \cos^2 x$

$0 = 2\cos^2 x - 3\sin x$

$0 = 2(1 - \sin^2 x) - 3\sin x$

$0 = 2 - 2\sin^2 x - 3\sin x$

$2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x + 4)(2\sin x - 1) = 0$

$(\sin x + 2)(2\sin x - 1) = 0$

$\sin x = -2$ $2\sin x - 1 = 0$

                $\sin x = \frac{1}{2}$

$\boxed{x = 30° + 360k}$

$\boxed{150° + 360k}$

15) $\cos 2x + 5 \cos x = 2$

$2\cos^2 x - 1 + 5\cos x - 2 = 0$

$2\cos^2 x + 5\cos x - 3 = 0$

$(2\cos x + 6)(2\cos x - 1) = 0$

$(\cos x + 3)(2\cos x - 1) = 0$

$\cos x = -3$ $\cos x = \frac{1}{2}$

$x = 60° + 360k$

$= 300° + 360k$

16) $\sqrt{3} \tan x + 1 = 0$

$\sqrt{3}\tan x = -1$

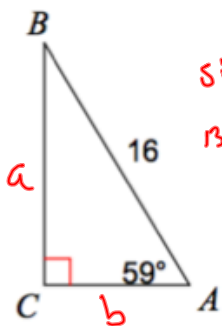$\tan x = \frac{-1}{\sqrt{3}}$

$\tan x = \frac{-\sqrt{3}}{3}$

$\boxed{x = 150° + 180k}$

Directions: Solve the triangle.

16)

B

16

a

59°

C   b   A

$\sin 59 = \frac{a}{16}$

$\cos 59 = \frac{b}{16}$

$13.7 = a$

$8.2 = b$

$\angle C = 31°$

17)

A

13

b

B   10   C
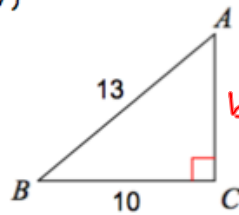
$13^2 = 10^2 + b^2$

$169 - 100 = b^2$

$\sqrt{69} = \sqrt{b^2}$

$8.3 = b$

$\sin A - \frac{10}{13}$

$A = 50.3°$

$\cos B = \frac{10}{13}$

$B = 39.7°$